

# Serie 02 - Solution

## Preamble

### Dopant Ionization: Thermal Dependency

During the course, you have learned that the electron and hole density is obtained by integrating the probability of carriers having a certain energy, described by the Fermi-Dirac distribution function, with the density of states for that energy.

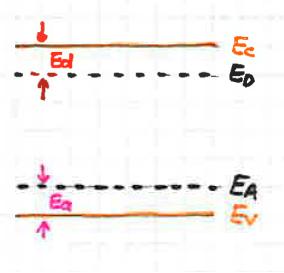


Figure 1: Representation of donor and acceptor states in the energy band diagram of a semiconductor.

From an energy perspective, a donor or acceptor is an atom with an ionization energy located in the forbidden band but near the conduction or valence level. A donor typically has an ionization energy  $E_D$  close to the conduction band. For a reasonable doping level  $N_D$ , the density of states provided by the dopant can be approximated as a Dirac function at  $E_D$  multiplied by the dopant concentration. Additionally, we need to account for the donor degeneracy factor, denoted as  $\beta_d$ , to consider the possibility of multiple ionization pathways. The Fermi-Dirac distribution function is still used to describe the probability of carriers having a certain energy. Therefore, the ionized donor concentration can be calculated as follows:

$$N_D^+ = \frac{N_D}{1 + \beta_d \cdot e^{\frac{E_f - E_D}{kT}}} \quad (1)$$

It is common to express the absolute difference between the conduction level and the ionization level as  $E_d$  for donors. A similar approach can be applied to acceptors, resulting in:

$$N_A^- = \frac{N_A}{1 + \beta_a \cdot e^{\frac{E_A - E_f}{kT}}} \quad (2)$$

Once again, it is common to express the absolute difference between the Valence level and the ionization level as  $E_a$  in this case.

## Given constants

$$k = 1.3806504 \cdot 10^{-23} [J/K]$$

$$q = 1.6021765 \cdot 10^{-19} [C]$$

## Exercise 01

Calculate the intrinsic carrier concentration of GaAs substrate at temperatures  $T_1 = 300 [K]$  and  $T_2 = 450 [K]$ . Given the conduction and valence band density of states at  $T_1$  as  $N_c = 4.7 \cdot 10^{17} [cm^{-3}]$  and  $N_v = 7 \cdot 10^{18} [cm^{-3}]$ , respectively, assuming they follow a thermal dependency law of  $T^{3/2}$ . For this exercise, we assume the band-gap of GaAs,  $E_g$ , to be temperature independent with a value of  $1.42 [eV]$ .

### Solution

As we have all the values, we will start by calculating the intrinsic carrier concentration of GaAs at  $T_1$ . For this purpose, we will use the following formula from the course:

$$n_i^2 = N_v N_c e^{\frac{-E_g}{kT}} \quad (3)$$

This formula is applicable to non-degenerate semiconductors, which means semiconductors with a Fermi level located inside the bandgap (which is always the case in this course). While this formula holds true, it is important to pay attention to the units used. In this case, we provide the Boltzmann constant in joules per Kelvin and the bandgap in electron volts. Therefore, you need to either convert the bandgap to joules or the Boltzmann constant to electron volts. In semiconductor physics, it is preferable to work with electron volts. We will convert the Boltzmann constant as follows:

$$k_{ev} = \frac{k_j}{q} \approx 86.2 \left[ \frac{\mu eV}{K} \right] \quad (4)$$

where  $q$  represents the elementary charge. And therefore:

$$n_i^2 = N_v N_c e^{\frac{-E_g}{kT_1}} \approx 4.59 \cdot 10^{12} [cm^{-6}] \quad (5)$$

$$n_i \approx 2.14 \cdot 10^6 [cm^{-3}] \quad (6)$$

In the exercise data, it is stated that  $N_c$  and  $N_v$  follow a thermal dependency law of  $T^{3/2}$ . Although this may not hold true for all temperatures, in the temperature range of interest, they can be expressed as:

$$N_{c/v}(T) = \alpha_{c/v} \cdot T^{\frac{3}{2}} \quad (7)$$

Since we know  $N_{c/v}(T_1)$  (given in the exercise), we can determine  $N_{c/v}(T_2)$ .

$$\alpha_{c/v} = \frac{N_{c/v}(T_1)}{T_1^{\frac{3}{2}}} \Rightarrow N_{c/v}(T_2) = N_{c/v}(T_1) \cdot \frac{T_2^{\frac{3}{2}}}{T_1^{\frac{3}{2}}} \quad (8)$$

Therefore:

$$N_c(T_2) N_v(T_2) = N_c(T_1) N_v(T_1) \cdot \left( \frac{T_2}{T_1} \right)^3 \approx 1.11 \cdot 10^{37} \text{ [cm}^{-6}] \quad (9)$$

and finally:

$$n_i^2 = N_c(T_1) N_v(T_1) \cdot \left( \frac{T_2}{T_1} \right)^3 e^{\frac{-E_g}{k_{ev}T_2}} \approx 1.39 \cdot 10^{21} \text{ [cm}^{-6}] \quad (10)$$

$$n_i \approx 3.72 \cdot 10^{10} \text{ [cm}^{-3}] \quad (11)$$

## Exercise 02

Calculate the concentration of holes at thermal equilibrium in a silicon sample at  $T_2 = 400 \text{ [K]}$  with a Fermi level above the valence band by approximately  $0.27 \text{ [eV]}$ . The valence band density of states of silicon is given at  $T_1 = 300 \text{ [K]}$  as  $N_v = 1.04 \cdot 10^{19} \text{ [cm}^{-3}]$ , and it follows a thermal dependency law of  $T^{3/2}$ .

### Solution

During the course, you learned that the hole density can be expressed as follows:

$$p = N_v(T) \cdot e^{-\frac{E_f - E_v}{kT}} \quad (12)$$

As previously assumed in **Exercise 01**,  $N_v$  follows a thermal dependency law of  $T^{3/2}$ , and we know  $N_v(T_1)$ . We apply a similar reasoning to find  $N_v(T_2)$ :

$$N_v(T_2) = N_v(T_1) \cdot \left( \frac{T_2}{T_1} \right)^{\frac{3}{2}} \approx 1.6 \cdot 10^{19} \text{ [cm}^{-3}] \quad (13)$$

In the exercise data, it is stated that the Fermi level is  $0.27 \text{ [eV]}$  above the valence band. Therefore:

$$E_f - E_v = 0.27 \text{ [eV]} \quad (14)$$

Finally, we just need to perform the numerical application of Eq. 12:

$$p = N_v(T) \cdot e^{-\frac{E_f - E_v}{kT}} \approx 6.35 \cdot 10^{15} \text{ [cm}^{-3}] \quad (15)$$

## Exercise 03

We consider a sample of silicon at  $T = 300 [K]$  with conduction band and valence band densities of states  $N_c = 2.8 \cdot 10^{19} [cm^{-3}]$  and  $N_v = 1.04 \cdot 10^{19} [cm^{-3}]$  at this temperature. We assume a Fermi level is lower than the conduction band by approximately  $0.25 [eV]$ , and the silicon band gap is  $E_g = 1.12 [eV]$ . Calculate the electron and hole densities, and compare them with the intrinsic density.

### Solution

During the course, you learned that the electron and hole density can be expressed as follows:

$$p = N_v e^{-\frac{E_f - E_v}{kT}} \quad (16)$$

$$n = N_c e^{-\frac{E_c - E_f}{kT}} \quad (17)$$

In the exercise data, it is stated that the Fermi level is  $0.25 [eV]$  below the

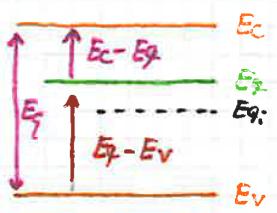


Figure 2: Representation of the energy band diagram.

conduction band. Therefore:

$$E_c - E_f = 0.25 [eV] \quad (18)$$

We also know that the band-gap can be defined as:

$$E_g = E_c - E_v = 1.12 [eV] \quad (19)$$

For this reason, we can find:

$$E_f - E_v = E_g - (E_c - E_f) = 0.87 [eV] \quad (20)$$

We just have to perform the numerical application of Eq. 16 with Eq. 20:

$$p = N_v e^{-\frac{E_f - E_v}{kT}} \approx 2.52 \cdot 10^4 [cm^{-3}] \quad (21)$$

We just have to perform the numerical application of Eq. 17 with Eq. 18:

$$n = N_c e^{-\frac{E_c - E_f}{kT}} \approx 1.77 \cdot 10^{15} [cm^{-3}] \quad (22)$$

## Exercise 04

We consider a p-type silicon sample doped with boron at a concentration of  $N_A = 10^{16} [\text{cm}^{-3}]$ . The energy difference between the valence level  $E_v$  and the acceptor ionization level  $E_A$  is  $E_a = 45 [\text{meV}]$ . The impurity level degeneracy factor is  $\beta_a = 4$  in this case. Find the temperature at which 90% of the acceptor impurities are ionized. For silicon at  $T = 300 [\text{K}]$  the conduction band densities of states  $N_c = 2.8 \cdot 10^{19} [\text{cm}^{-3}]$  and the valence band densities of states  $N_v = 1.04 \cdot 10^{19} [\text{cm}^{-3}]$ , and follows a thermal dependency law of  $T^{3/2}$ .

**Hint:** use the following approximation:

$$N_A - N_A^- = \frac{N_A}{1 + \frac{1}{\beta_a} \cdot e^{-\frac{E_f - E_A}{kT}}} \approx \beta_a N_A \cdot e^{-\frac{E_f - E_A}{kT}} \quad (23)$$

### Solution

Our first instinct is to rewrite Eq. 23 as the fraction of non-ionized dopants over the total dopant concentration and solve for it:

$$\frac{N_A - N_A^-}{N_A} = \frac{1}{1 + \frac{1}{\beta_a} \cdot e^{-\frac{E_f - E_A}{kT}}} \quad (24)$$

However, we quickly realize that we do not know the Fermi level. Therefore, we need to find a mathematical approach to extract it. In this case, since 90% of the acceptors are ionized:

$$N_A^- = N_a \cdot 0.9 = 9 \cdot 10^{15} \quad (25)$$

We can use the approximation developed in the **To Go Further** subsection of **Exercise 01** in the solution of **Series 01** to determine the hole carrier concentration  $p_0$ :

$$N_A^- \gg ni \quad \Rightarrow \quad p_0 \approx N_A^- \quad (26)$$

We can now express  $N_A$  as follows:

$$p_0 + N_A - N_A^- \approx N_A \quad (27)$$

Since 90% of the acceptors are ionized, 10% remain non-ionized. We now rewrite the fraction of non-ionized dopants from Eq. 24:

$$\frac{N_A - N_A^-}{p_0 + N_A - N_A^-} = 0.1 \quad (28)$$

Additionally, we know that  $p_0$  can be expressed as:

$$p_0 = N_v (T) \cdot e^{-\frac{E_f - E_v}{kT}} \quad (29)$$

Since only 10% of the dopants are not ionized, we can justify (even if somewhat generously) the approximation proposed in the hint of this exercise Eq. 23:

$$\frac{N_A - N_A^-}{N_A} = \frac{1}{1 + \frac{1}{\beta_a} \cdot e^{\frac{E_f - E_A}{kT}}} = 0.1 \iff \frac{1}{\beta_a} \cdot e^{\frac{E_f - E_A}{kT}} = 9 \quad (30)$$

$$\frac{1}{\beta_a} \cdot e^{\frac{E_f - E_A}{kT}} \gg 1 \Rightarrow N_A - N_A^- \approx \beta_a N_A \cdot e^{-\frac{E_f - E_A}{kT}} \quad (31)$$

Now we rewrite Eq. 28 by substituting the results from Eq. 29 and Eq. 31:

$$\frac{N_A - N_A^-}{p_0 + N_A - N_A^-} = \frac{\beta_a N_A \cdot e^{-\frac{E_f - E_A}{kT}}}{N_v(T) \cdot e^{-\frac{E_f - E_v}{kT}} + \beta_a N_A \cdot e^{-\frac{E_f - E_A}{kT}}} \quad (32)$$

Dividing by the numerator:

$$\frac{N_A - N_A^-}{p_0 + N_A - N_A^-} = \frac{1}{\frac{N_v(T)}{\beta_a N_A} \cdot e^{-\frac{E_A - E_v}{kT}} + 1} = 0.1 \quad (33)$$

We have now eliminated the Fermi level from the equation. Next, we expand the valence band density of states  $N_v$  in terms of temperature, as done in previous exercises, using  $T_R$  as the reference room temperature where  $N_v$  is known:

$$\frac{N_A - N_A^-}{p_0 + N_A - N_A^-} = \frac{1}{\frac{N_v}{\beta_a N_A} \left( \frac{T}{T_R} \right)^{\frac{3}{2}} \cdot e^{-\frac{E_A - E_v}{kT}} + 1} = 0.1 \quad (34)$$

Despite all our efforts, this equation remains analytically unsolvable, and therefore, a numerical approach is used to determine the solution:

$$T \approx 193 [K] \quad (35)$$